

Nash Mills C of E Primary School

WRITTEN CALCULATION POLICY

Written: May 2014

Reviewed: December 2016

Reviewed:

Reviewed:

Rationale

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with curriculum September 2014. Through the policy, we aim to link key manipulatives and representations in order that the children can be vertically accelerated through each strand of calculation. We know that school wide policies, such as this, can ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, we would expect the majority of each class to be working at age-appropriate levels as set out in the National Curriculum 2014 and in line with school policy.

The importance of mental mathematics

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

To add and subtract successfully, children should be able to:

- ·Recall all addition pairs to 9 + 9 and number bonds to 10
- · Recognise addition and subtraction as inverse operations
- · Add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
- \cdot Add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 70)
- \cdot Partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)
- · Use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:

- · Add and subtract accurately and efficiently
- ·Recall multiplication facts to $12 \times 12 = 144$ and division facts to $144 \div 12 = 12$
- · Use multiplication and division facts to estimate how many times one number divides into another etc.
- ·Know the outcome of multiplying by 0 and by 1 and of dividing by 1.
- ·Understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
- · Recognise factor pairs of numbers (e.g. that $15 = 3 \times 5$, or that $40 = 10 \times 4$) and increasingly able to recognise common factors
- Derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)
- · Notice and recall with increasing fluency inverse facts
- · Partition numbers into 100s, 10s and 1s or multiple groupings
- · Understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- · Understand the effects of scaling by whole numbers and decimal numbers or fractions
- · Understand correspondence where n objects are related to m objects
- · Investigate and learn rules for divisibility

Progression in Addition and Subtraction

Addition and subtraction are connected.

Part Part Whole

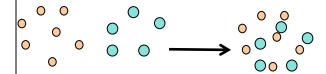
Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

Combining two sets (aggregation)

Putting together – two or more amounts or numbers are put together to make a total

ADDITION

7 + 5 = 12



Count one set, then the other set. Combine the sets and count again. Starting at 1.

Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1.



SUBTRACTION

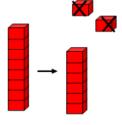
Taking away (separation model)

Where one quantity is taken away from another to calculate what is left.

8 - 2 = 6



Multilink towers- to physically take away objects

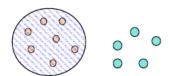


Combining two sets (augmentation)

This stage is essential in starting children to calculate rather than counting

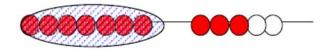
Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.

Counters:



Start with 7, then count on 8, 9, 10, 11, 12

Bead strings:



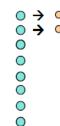
Make a set of 7 and a set of 5. Then count on from 7.

Finding the difference (comparison model)

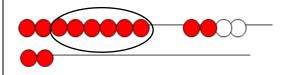
Two quantities are compared to find the difference.

8 - 2 = 6

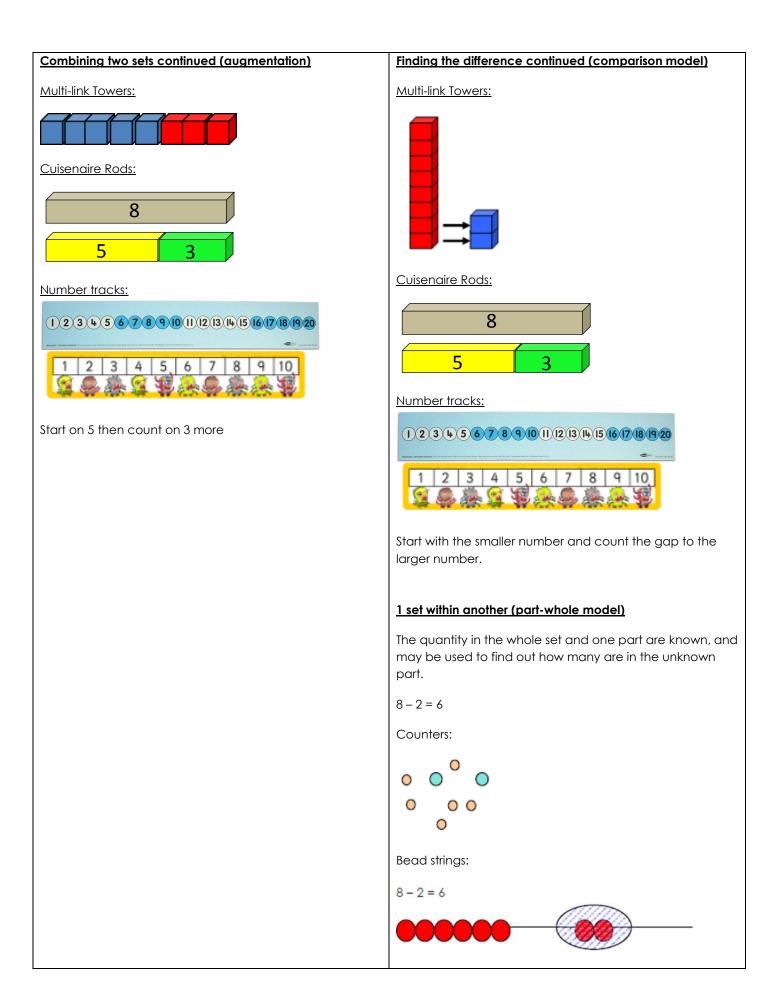
Counters:



Bead strings:



Make a set of 8 and a set of 2. Then count the gap.



Bridging through 10s

This stage encourages children to become more efficient and begin employ known facts.

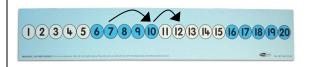
Bead string:



7 + 5 is decomposed/partitioned into 7 + 3 + 2.

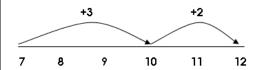
The bead sting illustrates 'how many more to the next multiple of 10?' (the children should identify how their number bonds are being applied) and then 'if we have used 3 of the 5 to get to 10, how many more do we need to add on? (ability to decompose/partition all numbers applied)

Number track:



Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number line



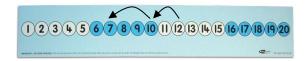
Bead string:



12-7 is decomposed/partitioned in 12-2-5.

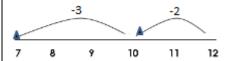
The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back? (ability to decompose/partition all numbers applied)

Number Track:



Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number Line:



Counting up or 'Shop keepers' method

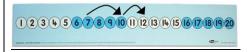
Bead string:



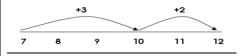
12-7 becomes 7 + 3 + 2.

Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'.

Number Track:



Number Line:



Compensation model (adding 9 and 11)

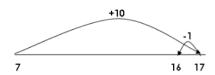
This model of calculation encourages efficiency and application of known facts (how to add ten)

7 + 9



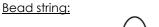
Children find 7, then add on 10 and then adjust by removing 1.

Number line:



This can then be extended to adding 19 and 21 when the children are ready.

18 – 9





Children find 18, then subtract 10 and then adjust by removing 1.

Number line:



This can then be extended to subtracting 19 and 21 when the children are ready.

Working with larger numbers

<u>10 + 10</u>

Ensure that the children have been transitioned onto Diennes and understand the abstract nature of the single 'tens' sticks and 'hundreds' blocks

Partitioning (Aggregation model)

34 + 23

Diennes:

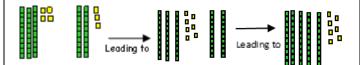


Children create the two sets with Diennes and then combine; ones with ones, tens with tens.

Partitioning (Augmentation model)

Diennes:

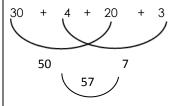
Encourage the children to begin counting from the first set of ones and tens, avoiding counting from 1. Beginning with the ones in preparation for formal columnar method.



Number line:



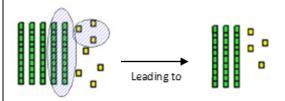
At this stage, children can begin to use an informal method to support, record and explain their method. (optional)



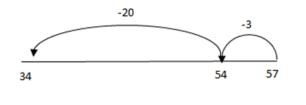
Take away (Separation model)

Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.

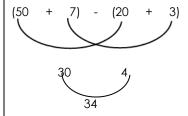
$$57 - 23 = 34$$



Number Line:



At this stage, children can began to use an informal method to support, record and explain their method (optional)

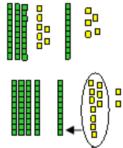


Using Diennes initially will help the children link a concrete material with numbers. Once this has been achieved number squares and number lines can be explored for addition and subtraction.

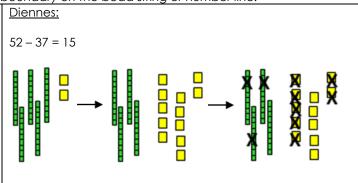
Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the units are greater than 10? Introduce the term 'exchange'. Using the Diennes equipment, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

<u>Diennes:</u> 37 + 15

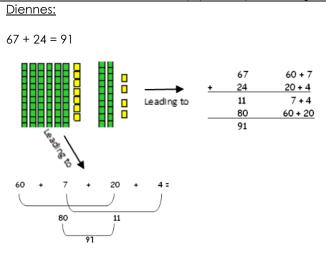


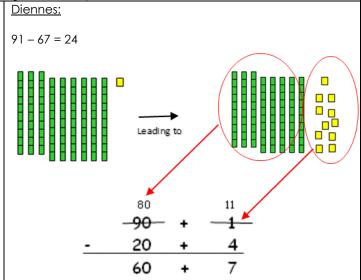
Discuss counting on from the larger number irrespective of the order of the calculation.

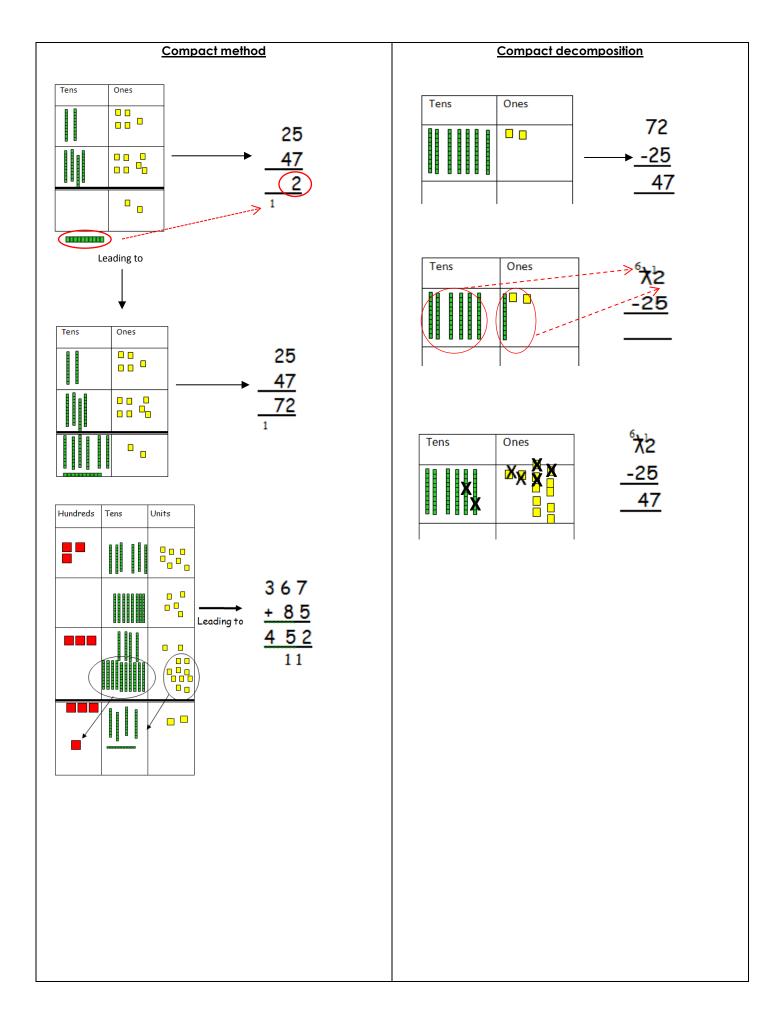


Expanded Vertical Method (optional)

Children are then introduced to the expanded vertical method to ensure that they make the link between using Diennes equipment, partitioning recording and the expanded vertical method.







Vertical Acceleration

By returning to earlier manipulative experiences we support children in make links across mathematics, encouraging 'If I know this...then I also know...' thinking.

Decimals

Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:



Each bead represents 0.1, each different block of colour equal to 1.0

Diennes:



0.1 1.0



10.0

Addition of decimals

Aggregation model of addition

counting both sets-starting at zero.

0.7 + 0.2 = 0.9

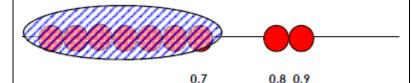


0.1 0.2 0.3 0.4 0.5 0.6 0.7

Augmentation model of addition:

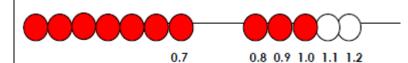
starting from the first set total, count on to the end of the second set.

0.7 + 0.2 = 0.9



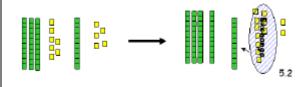
Bridging through 1.0

encouraging connections with number bonds. 0.7 + 0.5 = 1.2



Partitioning

3.7 + 1.5 = 5.2

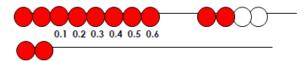


Subtraction of decimals Take away model

0.9 - 0.2 = 0.7



0.8 - 0.2 =



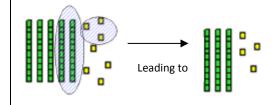
Bridging through 1.0

encourage efficient partitioning. 1.2 - 0.5 = 1.2 - 0.2 - 0.3 = 0.7



Partitioning

5.7 - 2.3 = 3.4



Gradation of difficulty- addition

- 1. No exchange
- 2. Extra digit in the answer
- 3. Exchanging units to tens
- 4. Exchanging tens to hundreds
- 5. Exchanging units to tens and tens to hundreds
- 6. More than two numbers in calculation
- 7. As 6 but with different number of digits
- 8. Decimals up to 2 decimal places (same number of decimal places)
- 9. Add two or more decimals with a range of decimal places.

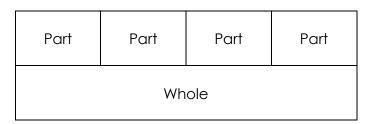
Gradation of difficulty- subtraction

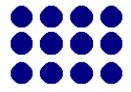
- 1. No exchange
- 2. Fewer digits in the answer
- 3. Exchanging tens for units
- 4. Exchanging hundreds for tens
- 5. Exchanging hundreds to tens and tens to units
- 7. As 6 but with different number of digits
- 8. Decimals up to 2 decimal places (same number of decimal places)
- 9. Subtract two or more decimals with a range of decimal places.

Progression in Multiplication and Division

Multiplication and division are connected.

Both express the relationship between a number of equal parts and the whole.





The following array, consisting of four columns and three rows, could be used to represent the number sentences: -

$$3 \times 4 = 12$$
,

$$4 \times 3 = 12$$
,

$$3 + 3 + 3 + 3 = 12$$
,

$$4 + 4 + 4 = 12$$
.

And it is also a model for division

$$12 \div 4 = 3$$

$$12 \div 3 = 4$$

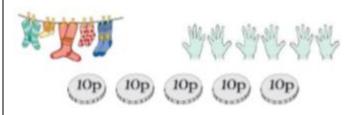
$$12 - 4 - 4 - 4 = 0$$

$$12 - 3 - 3 - 3 - 3 = 0$$

MULTIPLICATION

Early experiences

Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.



DIVISION

Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.



Repeated addition (repeated aggregation)

3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5×3

Children learn that repeated addition can be shown on a number line.



Children learn that repeated addition can be shown on a bead string.

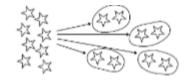


Children also learn to partition totals into equal trains using Cuisenaire Rods



Sharing equally

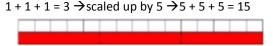
6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.



<u>Scaling</u> (Difficulty of scaling will progress through the school)

This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles.

This can be written as:



For example, find a ribbon that is 4 times as long as the blue ribbon.



Upper KS2 - We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to 3 \times 0.5 = 1.5.

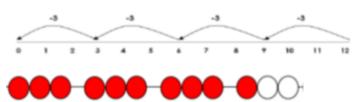
Grouping or repeated subtraction

There are 6 sweets. How many people can have 2 sweets each?



Repeated subtraction using a bead string or number line

 $12 \div 3 = 4$



The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as 'how many 3s make 12?'

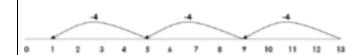
Cuisenaire Rods also help children to interpret division calculations.



Grouping involving remainders

Children move onto calculations involving remainders.

$$13 \div 4 = 3 r1$$



Or using a bead string see above.

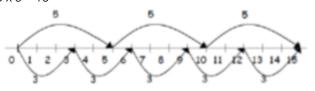
<u>Commutative</u> (Commutative Law is the law that says you can swap numbers around and still get the same answer when you add or multiply)

Children learn that 3×5 has the same total as 5×3 .

This can also be shown on the number line.

$$3 \times 5 = 15$$

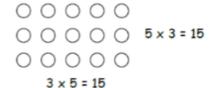
$$5 \times 3 = 15$$



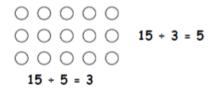
Children learn that division is **not** commutative and link this to subtraction.

Arrays

Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number.



Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities.



Inverse operations

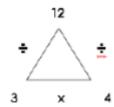
Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

$$3 \times 4 = 12$$

$$4 \times 3 = 12$$

$$12 \div 3 = 4$$

$$12 \div 4 = 3$$

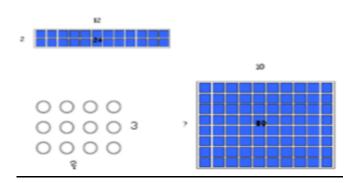


Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

$$\square$$
 x 5 = 20 3 x \triangle = 18 0 x \square = 32

$$24 \div 2 = \Box \quad 15 \div O = 3 \quad \Delta \div 10 = 8$$

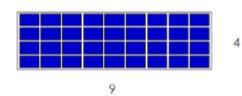
This can also be supported using arrays: e.g. 3 X ? = 12



Partitioning for multiplication

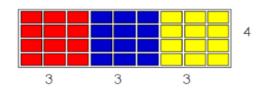
Arrays are also useful to help children visualise how to partition larger numbers into more useful arrays.

$$9 \times 4 = 36$$



Children should be encouraged to be flexible with how they use number and can be encouraged to break the array into more manageable chunks.

9 x 4 =

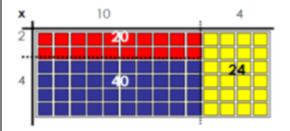


Which could also be seen as

$$9x 4 = (3 \times 4) + (3 \times 4) + (3 \times 4) = 12 + 12 + 12 = 36$$

Or
$$3 \times (3x4) = 36$$

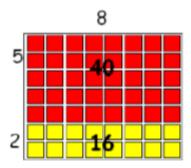
And so $6 \times 14 = (2 \times 10) + (4 \times 10) + (4 \times 6) = 20 + 40 + 24 = 84$



Partitioning for division

The array is also a flexible model for division of larger numbers

$$56 \div 8 = 7$$



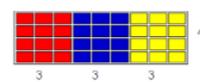
Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between division and multiplication.

$$56 \div 8 = (40 \div 8) + (16 \div 8) = 5 + 2 = 7$$

To be successful in calculation learners must have plenty of experiences of being flexible with partitioning, as this is the basis of distributive and associative law.

Associative law (multiplication only)

E.g.
$$3 \times (3 \times 4) = 36$$



The principle that if there are three numbers to multiply these can be multiplied in any order.

Distributive law (multiplication):-

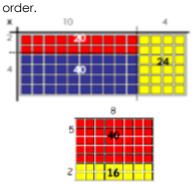
E.g.
$$6 \times 14 = (2 \times 10) + (4 \times 10) + (4 \times 6) = 20 + 40 + 24 = 84$$

This law allows you to distribute a multiplication across an addition or subtraction.

Distributive law (division):-

E.g.
$$56 \div 8 = (40 \div 8) + (16 \div 8) = 5 + 2 = 7$$

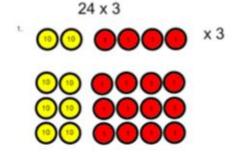
This law allows you to distribute a division across an addition or subtraction.



Arrays leading into the grid method

Children continue to use arrays and partitioning where appropriate, to prepare them for the grid method of multiplication.

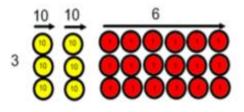
Arrays can be represented as 'grids' in a shorthand version and by using place value counters we can show multiples of ten, hundred etc.



<u>Arrays leading into chunking and then long and short division</u>

Children continue to use arrays and partitioning where appropriate, to prepare them for the 'chunking' and short method of division. Arrays are represented as 'grids' as a shorthand version.

e.g.
$$78 \div 3 =$$

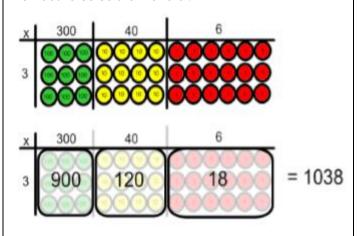


$$78 \div 3 = (30 \div 3) + (30 \div 3) + (18 \div 3) =$$

$$10 + 10 + 6 = 26$$

Grid method

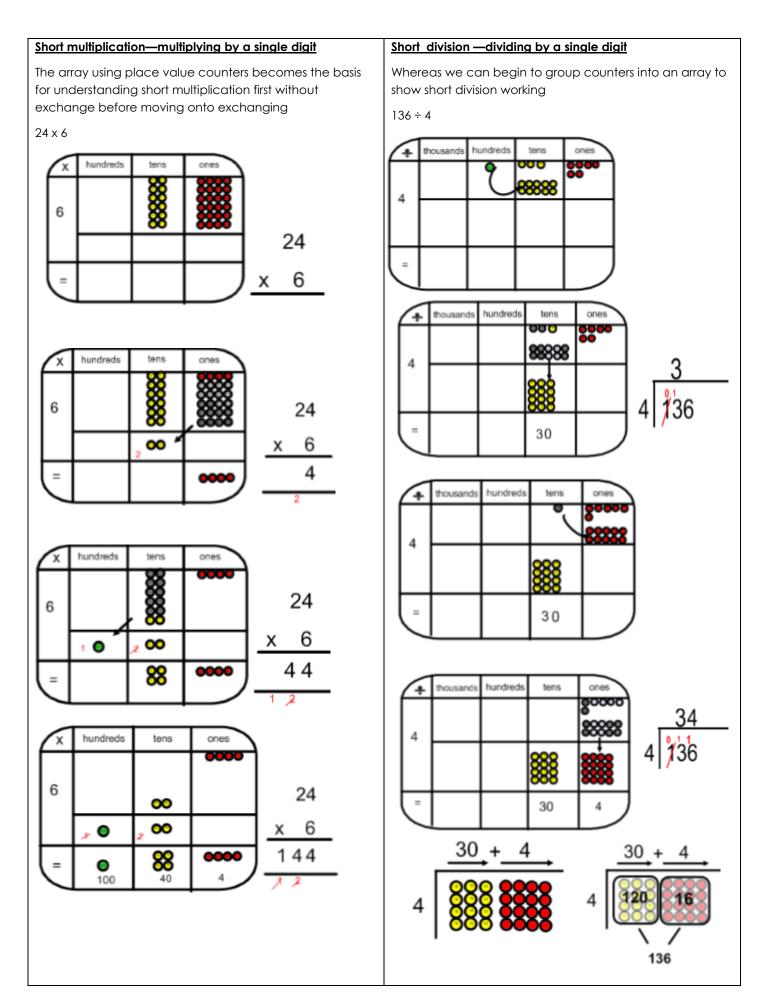
This written strategy is introduced for the multiplication of TU x U to begin with. It may require column addition methods to calculate the total.



The vertical method- 'chunking' leading to long division

See above for example of how this can be modelled as an array using place value counters.

So
$$78 \div 3 = 10 + 10 + 6 = 26$$



Gradation of difficulty (Short multiplication)

- 1. TO x O no exchange
- 2. TO x O extra digit in the answer
- 3. TO x O with exchange of ones into tens
- 4. HTO x O no exchange
- 5. HTO x O with exchange of ones into tens
- 6. HTO x O with exchange of tens into hundreds
- 7. HTO x O with exchange of ones into tens and tens into hundreds
- 8. As 4-7 but with greater number digits x O
- 9. O.t x O no exchange
- 10. O.t with exchange of tenths to ones
- 11. As 9 10 but with greater number of digits which may include a range of decimal places x O

Gradation of difficulty (Short division)

- 1. TO ÷ O no exchange no remainder
- 2. TO ÷ O no exchange with remainder
- 3. TO ÷ O with exchange no remainder
- 4. TO ÷ O with exchange, with remainder
- 5. Zeroes in the quotient e.g. $816 \div 4 = 204$
- 6. As 1-5 HTO ÷ O
- 7. As 1-5 greater number of digits ÷ O
- 8. As 1-5 with a decimal dividend e.g. $7.5 \div 5$ or $0.12 \div 3$
- 9. Where the divisor is a two digit number

See below for gradation of difficulty with remainders

Dealing with remainders

Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly.

e.g.

- ·I have 62p. How many 8p sweets can I buy?
- · Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?

Gradation of difficulty for expressing remainders

- 1. Whole number remainder
- 2. Remainder expressed as a fraction of the divisor
- 3. Remainder expressed as a simplified fraction
- 4. Remainder expressed as a decimal

Long multiplication—multiplying by more than one digit

Children will refer back to grid method and compare before being required to record as —

Methods below taken from appendix of curriculum 2014

24 × 16 becomes 24 × 32 becomes

Answer: 384

Answer: 768

124 × 26 becomes

Answer: 3 224

Long division —dividing by more than one digit

Children should be reminded about partitioning numbers into multiples of 10, 100 etc. before recording as—

Methods below taken from appendix of curriculum 2014 – this could link strongly to 'chunking' as a method of long division.

432 ÷ 15 becomes

432 ÷ 15 becomes

Answer: 28 remainder 12 Answer: 28 ⁴/₅

432 ÷ 15 becomes

Answer: 28.8

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